What is the focus and goals of the textbook?

Special Relativity, Electrodynamics and General Relativity: From Newton to Einstein

1. Special Relativity with a Mission

This book is a revision and extension of “Introduction to Relativity”. That book presents an approach to Relativity in which the theory is developed through “thought experiments” that illustrate the concepts in simple fashions. For example, in order to derive time dilation and Lorentz contraction, a clock is constructed out of mirrors and a light beam, and we learn how it works in its rest frame and in a frame in which it moves with velocity $v$.

This analysis is simple and direct because light beams and rods are governed by the postulates of Special Relativity:

1. The laws of physics are the same in all inertial frames of reference.
2. There is a common finite speed limit $c$ in all inertial frames.

Throughout the book the rules of non-relativistic physics (Newton’s World) are contrasted with relativistic physics (Einstein’s World). It is the second postulate that distinguishes Newtonian rules from Einstein’s. In Newton’s World velocities are unbounded but in Einstein’s World the
speed of light \( c \) is an upper bound. The existence of a speed limit combined with Postulate 1, that all inertial frames are equivalent, produces relativistic space-time.

The book shows that time dilation, Lorentz contraction and the Relativity of Simultaneity, that clocks which are separated in space and synchronized in one frame are not synchronized in a frame in relative motion, are intimately related.

**The book builds the subject from the ground up.** It uses Minkowski spacetime diagrams, which show the spatial and temporal coordinates of two frames in relative motion \( v \). Time dilation, Lorentz contraction, and the Relativity of Simultaneity can be understood through simple pictures (cartoons, really), both qualitatively and quantitatively. From this perspective, we study the Twin Paradox and see that there is nothing really paradoxical about it—when one twin leaves the other and takes a round trip, she returns younger than her sibling. It is fun to see how it works out!

Once we have mastered kinematics—the measurement of space and time intervals—we move to dynamics, the study of energy, momentum, interactions and equations of motion. In Newtonian mechanics there is momentum conservation, on the one hand, and mass conservation, on the other. In relativity, however, once we have figured out how spatial and temporal measurements are related in inertial frames, the properties of momentum, energy, and mass are determined. We present another of Einstein's famous thought experiments that shows that energy and inertia (mass) are two aspects of one concept, relativistic energy, and find \( E = mc^2 \). The properties of relativistic momentum then follow from the properties of spatial and temporal measurements and we learn that momentum and energy conservation must go hand-in-hand.

Since Newton's Second Law, force equals mass time acceleration, is not consistent with the existence of a speed limit, we must modify it and invent a new force law that is relativistic. We do this aided, again, by the original thought experiments of the masters, Einstein and Max Born.
In this case, a close look at an inelastic collision in several frames of reference leads us to relativistic momenta and energy. From there we obtain the relativistic version of Newton's Second Law. We then consider high-energy collisions between particles and illustrate how energy can be converted into mass and how mass can be converted into energy. These illustrations show that Newton’s concept of the integrity of a particle is violated in Relativity. There is no way in Relativity to restrict one’s attention to the “single particle sector”. The fundamentals of Relativity simply don’t allow it. This fact has consequences beyond the scope of this book and finds its fruition in relativistic field theories of particle physics.

In the first edition of this book we were satisfied with introductory work on Relativistic dynamics. In this extended book we have greater ambitions. We will start with electrostatics, that there is a static force which falls off as the square of the distance between charges. We will use this fact coupled with Special Relativity to deduce:

1. The existence of magnetism, a field that is generated by electric currents,
2. The Lorentz Force Law, which states that particles are deflected by electric fields and also by magnetic fields in which case the force is proportion to the particle’s velocity,
3. Maxwell’s Equations: the Ampere-Maxwell equation predicting how currents produce magnetic fields and Faraday’s Law which predicts how changing magnetic fields produce electric forces (electromotive forces),
4. The Wave Equation for Electromagnetism and the demonstration that light travels at the speed limit c of Special Relativity in all inertial reference frames.

The book emphasizes that magnetism is a consequence of Relativity--that Newton’s World, a world without a speed limit, has no place for this phenomenon. In addition, magnetism produces velocity dependent forces on charged particles that do not satisfy Newton’s Third Law, that action
equals reaction. Once we understand that Newton’s Third Law fails, we must look elsewhere for an understanding of momentum conservation in dynamical interactions.

Once we have derived the Lorentz Force Law, we are able to deduce how electric and magnetic fields transform under boosts. We learn from carrying out this exercise that electric and magnetic fields mix under transformations between frames of reference. These effects are consequences of the fact that space and time mix under such transformations. These results illustrate one of Einstein’s famous remarks that “electric forces in one frame can be interpreted as magnetic effects in another”. The problem sets in this part of the book consider more traditional ways of obtaining the same results, considering capacitors and solenoids in various reference frames.

These arguments elucidate the nature of magnetism, but they don’t quite get to the heart of the matter. Electrostatics states that charges are the sources of electric fields. This is stated quantitatively through Gauss’ Law. On the other hand, we learn that electric currents are the sources of magnetic fields. Since electric currents are produced from charge densities by Lorentz Transformations (“boosts”), it must be that the equation that predicts that magnetic fields are produced from electric currents can be obtained by boosting Gauss’ Law. We carry out this exercise and discover Ampere-Maxwell’s equation. This is Ampere’s Law corrected for non-stationary currents by Maxwell’s Displacement current. Similarly we obtain Faraday’s Law from the divergence-free character of magnetic fields.

This book shows the student that a scientist can build the theory of Electromagnetism from electrostatics and special relativity. Unlike conventional textbooks we learn to manipulate differential equations and accomplish our goals in general fundamental terms rather than working
through myriad illustrations of devices. Once we have Maxwell’s equations it is an easy task to show that relativity elevates electromagnetic fields to independent dynamical degrees of freedom which propagate at the speed limit $c$.

In this journey we find it necessary to forsake Newtonian ideas and replace them with modern concepts: forces lead to fields which become dynamical objects that propagate, non-local conservation laws such as Newton’s Law of action-reaction are thrown over and are replaced by local conservation laws which are compatible with the relativistic notion of causality and the finiteness of the speed of propagation of information. In addition, the integrity of particle number, so crucial in Newton’s World, is replaced by particle creation and destruction which is forced upon us by the energy-mass relationship of relativity, etc.

Thus our introduction to electrodynamics is an alternative to the traditional approach popularized by the textbooks of J. D. Jackson [1] and his many followers. This book starts with Postulates 1 and 2 of Special Relativity and the primitive notion that charges are the local sources of electric fields, and develops the fundamentals of electromagnetism: unified magnetic and electric fields, the Lorentz Force Law, Maxwell’s Equations and the Wave Equation. After mastering these fundamentals the student is in a position to do more complex applications of relativity and electrodynamics. The approach in this book is simple, and straight-forward. The derivation of Maxwell’s Equations, having come from such humble beginnings in Chapter 1, should be an epiphany for the student.
2. **General Relativity**

Our last topic in this book is the General Theory of Relativity, where we consider accelerated reference frames in Einstein's world. The key insight here is Einstein's version of the Equivalence Principle: there is no physical means to distinguish a uniform gravitational field from an accelerated reference frame.

This principle has many interesting forms and applications. Suppose you are on the surface of Earth and want to understand the influence of the gravitational field on your measuring sticks and clocks compared to those of your assistant who is at a greater height in the Empire State Building. Einstein suggests that the assistant jump out the window, because in a freely falling frame all effects of gravity are eliminated and we have a perfectly inertial environment where special relativity holds to arbitrary precision! During his descent, your assistant can make measurements of clocks and meter sticks fixed at various heights along the building and measure how their operation depends on their gravitational potential. We pursue ideas like this one in the book to derive the gravitational red-shift, the fact that clocks close to stars run more slowly than those far away from stars; the resolution of the Twin Paradox as a problem in accelerating reference frames; and the bending of light by gravitational fields.

A fascinating aspect of the Equivalence Principle is its universality, which becomes particularly clear when we calculate the bending of light as a ray glances by the Sun. Why does the light ray feel the presence of the mass of the Sun? The Equivalence Principle states that an environment with a gravitational field is equivalent to an environment in an accelerating reference frame—explicit acceleration clearly affects the trajectory of light and any other physical phenomena. So gravity becomes a problem in accelerated reference frames, which is just a problem in coordinate transformations, which is an aspect of geometry! We show that this problem in
geometry must be done in the context of four-dimensional space–time, our world of Minkowski diagrams. Einstein's theory of gravity brought modern geometry, the study of curved spaces, into physics forever.

As long as we concentrate on gravitational fields of ordinary strength, we are able to use the Equivalence Principle to make reliable, accurate predictions. The Equivalence Principle reduces gravity to an apparent force, much like the centrifugal or Coriolis forces that we feel when riding on a carousel. In fact, we use relativistic turntables and rotating reference frames as an aide to studying and deriving relativistic gravitational effects.

However, we are also interested in strong gravity, situations where the Equivalence Principle is not sufficient to describe the physics. We discuss tidal forces which are at the heart of the dynamics of general relativity. This is done both within Newton’s world and general relativity where we learn that tidal forces and the curvature of spacetime are two sides of the same coin. Since curvature is the essence of the dynamics of general relativity, we review classical differential geometry and learn about the curvature of curves and the curvature of two dimensional surfaces. Gauss’ curvature is introduced and the fundamental theorems of classical differential geometry are derived and discussed.

These developments lead to a discussion of modern general relativity. The critical symmetries of special relativity were Lorentz transformations (boosts) and in general relativity these symmetries are elevated to local symmetries, general coordinate transformations. The Riemann tensor is introduced as a measure of the local, intrinsic curvature of spacetime. These ideas lead to the central idea of general relativity, that local energy-momentum produce local spacetime curvature. This idea is expressed quantitatively through Einstein’s field equation which is introduced as a grand extension of Newton’s equation for the gravitational potential. We solve
these equations in the case of a static, symmetric mass distribution which gives rise to the Schwarzschild metric. This metric contains a black hole which we study in considerable detail. We find that there is an “event horizon”, the Schwarzschild radius, inside of which there is no escape from the gravity. We also consider the production and propagation of gravitational waves. We see that gravity waves are described by fluctuations of the spacetime metric that travel at the speed limit of special relativity and are generated by oscillating quadrupole moments of mass distributions. We review the “Advanced LIGO” experiment that discovered gravity waves generated from the violent merger of two black holes over one billion light years away from earth.

We end our discussion of general relativity by comparing its fundamentals with modern theories of elementary particles, which are also based on local symmetry groups, as well as a look at current puzzles and unsolved problems, such as Dark Energy.

3. Background Reading and Recommendations for Future Reading

Although the perspective of this book is its own, it owes much to other presentations. The influence of C. P. French's 1968 book [2] *Special Relativity* is considerable, and references throughout the text indicate where my discussions follow his. To my knowledge, this is the finest textbook written on the subject because it balances theory and experiment perfectly. The reader will find discussions of the Michelson–Morley experiment and early tests of relativity there. French's discussions of energy and momentum are reflected in my later chapters, and his problem sets are a significant influence on those included here. The present book goes beyond French’s when we derive the Lorentz transformation laws for the electric and magnetic field and then deduce all of Maxwell’s Equations by boosting Gauss’ Law. We include a shortened version of French’s demonstration that the magnetic attraction between parallel current carrying wires is due to a
minuscule Lorentz contraction effect. We show that the force between such wires is strong enough to bend typical cooper wires because the huge number of electrons in the wire multiplies the tiny Lorentz contraction they experience due to their tiny velocities relative to the wire’s stationary Cu+ ions. This observation explains why magnetic forces can be substantial under ordinary lab conditions even though magnetism is a pure relativistic effect. These numbers underlie how relativity was discovered at the turn of the 20th century, long before the modern era of particle accelerators where particle velocities close to the speed limit were possible.

Another influential book is “The Classical Theory of Fields” by L. D. Landau and E. M. Lifshitz [3]. Unlike most books on electromagnetism which follow the historical development of the field and introduce relativity towards the end, Landau and Lifshitz start with relativity and build the theory of relativistic particles and light. The emphasis is on fundamental principles rather than applications. Unfortunately the book is aimed at advanced students and most editions are flush with typographical errors. Nonetheless, the book is very inspiring and has unique insights into the subject. Several of the more challenging problems in chapters 7-10 in this volume were inspired by Landau and Lifshitz. This book also introduces general relativity and includes derivations of the Schwarzschild metric, gravitational waves, the bending of light rays in a gravitational field, etc. The discussions are brief and powerful, the essence of of Landau’s brilliant approach to physics.

The exposition by N. D. Mermin [4] influenced several discussions of the paradoxes of special relativity. This book is also recommended to the student because it shows a condensed-matter physicist learning the subject and finding a comfort level in it through thought-provoking analyses that avoid lengthy algebraic developments. The huge book by J. A. Wheeler and E. F. Taylor [5] titled *Spacetime Physics* inspired several of our discussions and problem sets. This
book, a work that only the unique, creative soul of John Archibald Wheeler could produce, is recommended for its leisurely, interactive, thought-provoking character. Finally, the books by W. Rindler [6, 7], a pioneer in modern general relativity, are also recommended. His book *Essential Relativity* [7] is a solid introduction to general relativity rooted in the era of Einstein and the pioneers. After the student has mastered electricity and magnetism and Lagrangian mechanics, he or she could tackle “The Classical Theory of Fields” by L. D. Landau and E. M. Lifshitz [3] and *Essential Relativity* [7].

The future of research in relativity and field theory is bright. Hopefully this book will spark some interest in its future practitioners.

“Time travels in diverse paces with diverse persons”, Rosalind, from Act 3, Scene 2, “As You Like It” by William Shakespeare.

“Mathematics allows you to exceed your imagination”, Lev Landau, Moscow, 1952.

“Magic!”, student, anonymous, 2018

References


